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Hyperbolic heat conduction with surface radiation and reflection

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INTRODUCTION

THE WAVE nature of heat propagation prevails at very low temperatures or at very early times in highly transient heat transfer processes. Since Vernotte [1] built the hyperbolic equation model for the heat conduction with wave nature, hyperbolic heat conduction has been the subject of many studies. Recently, two sets of works have been added to the extensive literature for hyperbolic heat conduction. One showed the existence of the reflections at the end wall [2, 3] and the other considered the effects of surface radiation on hyperbolic heat conduction in a semi-infinite medium [4–6]. Although some authors [7–11] have analytically studied the reflections of a heat wave, they have not considered the effects of surface radiation. Therefore, the purpose of this work is to study the propagation of a heat wave in a finite slab with a radiating surface. The other surface of the slab is kept isothermal or insulated. Numerical computations are performed to examine the propagation of the heat wave, especially the reflection of the wave at the latter boundary. Temperature distributions for such situations are found by an integral equation method [6], which can handle the wave front exactly.

ANALYSIS

This work is concerned with the hyperbolic heat conduction in a finite slab with constant properties. Initially the slab is at zero temperature, and for $t > 0$ a heat flux is applied at the boundary surface $x = 0$. While the boundary surface $x = 0$ dissipates heat by radiation into the surroundings at zero temperature, the boundary surface $x = L$ is under applied zero temperature or insulated. The dimensionless formulation of this problem is given by

$$\frac{\partial \theta}{\partial \xi}+\frac{\partial Q}{\partial \eta}=0 \quad \text{in} \quad 0<\eta<1 \quad \text{for} \quad \xi>0 \tag{1}$$

$$\frac{\partial Q}{\partial \xi}+\frac{\partial \theta}{\partial \eta}+2Q=0 \quad \text{in} \quad 0<\eta<1 \quad \text{for} \quad \xi>0 \tag{2}$$

$$\theta(\eta,0)=0 \tag{3}$$

$$Q(\eta,0)=0 \tag{4}$$

$$Q(0,\xi)=-\frac{\alpha_s}{N}\theta^4(0,\xi)+G(\xi) \tag{5}$$

$$\theta(\eta_L,\xi)=0 \tag{6a}$$

or

$$\frac{\partial \theta}{\partial \eta}(\eta_L,\xi)=0 \tag{6b}$$

where

$$\xi=\frac{c^2t}{2\alpha}, \quad \eta=\frac{cx}{2\alpha} \tag{7a,b}$$

$$\theta(\eta,\xi)=\frac{T(x,t)-T(x,0)}{\alpha f_r/kc}, \quad Q(\eta,\xi)=\frac{q(x,t)}{f_r} \tag{8a,b}$$

$$N=\frac{k^4c^4}{n^2\sigma\alpha^4 f_r^3}. \tag{9}$$

Here, α_s denotes the surface absorptivity, $G(\xi)$ the dimensionless given flux at $\eta = 0$, η_L the dimensionless slab thickness, c the velocity of propagation of the thermal wave in the medium, α the thermal diffusivity, T the temperature, f_r a reference heat flux, q the heat flux, k the thermal conductivity, n the refractive index of the medium and σ the Stefan–Boltzmann constant. In this work, both η_L and $G(\xi)$ are taken as unity.

Laplace transforming in ξ, solving the resulting boundary value problem and applying the convolution theorem in the inverse transform yields the analytical solution for the temperature distribution in the form

$$\begin{aligned}\theta(\eta,\xi)=\int_0^\xi e^{-\tau}\Bigg\{&\sum_{m=1}^{\infty} B\Bigg[I_0(\lambda_1)U(\tau-2m+\eta)\\&+\frac{\tau}{\lambda_1}I_1(\lambda_1)U(\tau-2m+\eta)+I_0(\lambda_1)\delta(\tau-2m+\eta)\Bigg]\\&+\sum_{m=0}^{\infty} B\Bigg[I_0(\lambda_2)U(\tau-2m-\eta)\\&+\frac{\tau}{\lambda_2}I_1(\lambda_2)U(\tau-2m-\eta)\\&+I_0(\lambda_2)\delta(\tau-2m-\eta)\Bigg]\Bigg\}\Bigg[-\frac{\alpha_s}{N}\theta^4(0,\xi-\tau)+1\Bigg]d\tau\end{aligned} \tag{10a}$$

where

$$\lambda_1=\sqrt{(\tau^2-(2m-\eta)^2)}, \quad \lambda_2=\sqrt{(\tau^2-(2m+\eta)^2)} \tag{10b,c}$$

$$B=\begin{cases}(-1)^m & \text{for the case with } \theta(1,\xi)=0\\ 1 & \text{for the case with } \dfrac{\partial\theta}{\partial\xi}(1,\xi)=0\end{cases} \tag{10d}$$

I_0 is the zeroth-order modified Bessel function, I_1 the first-order modified Bessel function, and U the unit step function. Equation (10a) is an exact solution for $\theta(\eta,\xi)$ provided that the surface temperature $\theta(0,\xi)$ is available. Setting η in equation (10a) to zero yields the integral equation for the surface temperature

$$\begin{aligned}\theta(0,\xi)=\int_0^\xi e^{-\tau}\Bigg\{&I_0(\tau)U(\tau)+I_1(\tau)U(\tau)\\&+I_0(\tau)\delta(\tau)+2\sum_{m=1}^{\infty}B\Bigg[I_0(\lambda_3)U(\tau-2m)\\&+\frac{\tau}{\lambda_3}I_1(\lambda_3)U(\tau-2m)\\&+I_0(\lambda_3)\delta(\tau-2m)\Bigg]\Bigg\}\Bigg[-\frac{\alpha_s}{N}\theta^4(0,\xi-\tau)+1\Bigg]d\tau\end{aligned} \tag{11a}$$

where

$$\lambda_3=\sqrt{(\tau^2-(2m)^2)}. \tag{11b}$$

Equation (11a) is a non-linear Volterra integral equation

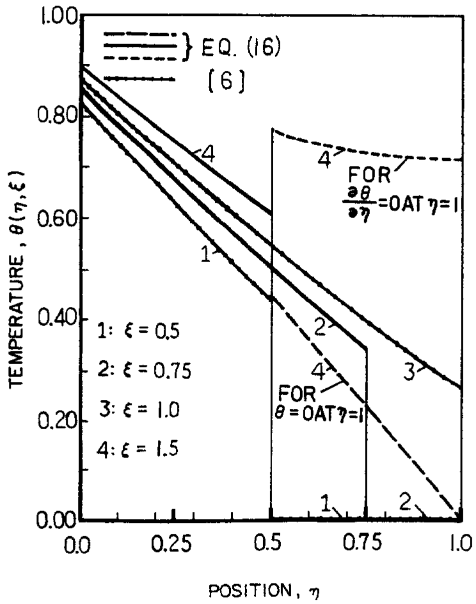

FIG. 1. Temperature distributions for a sequence of times ($\alpha_s/N = 1.0$).

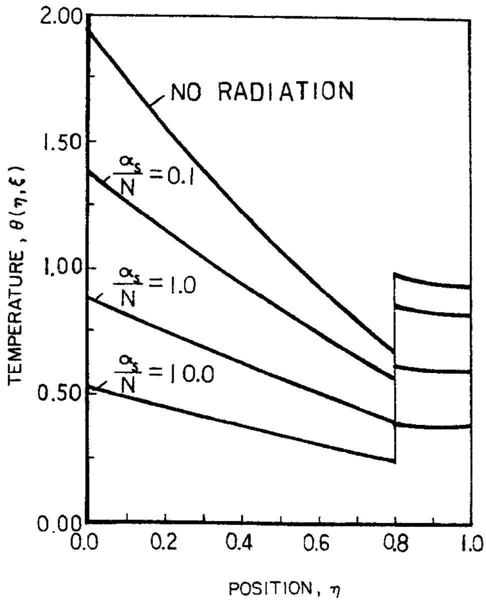

FIG. 3. Effects of surface radiation on temperature distributions at $\xi = 1.2$ in the medium with insulated boundary at $\eta = 1$.

of the second kind for $\theta(0, \xi)$. In practice, the time interval over which the major change of $\theta(0, \xi)$ occurs is finite. Thus equation (11a) can be solved by the method of successive approximation numerically. The details can be found elsewhere [6].

RESULTS AND DISCUSSION

Figure 1 shows the temperature distributions for a sequence of times in a slab subjected to the boundary conditions specified in equations (6a) and (6b). Because the heat wave has no knowledge of the boundary condition at $\eta = 1$ for $\xi < 1.0$, the temperature curves at $\eta < 1$ for $\xi = 0.5, 0.75, 1.0$ are the same as those for a semi-infinite slab, as shown in Fig. 1. The wave reverses its direction of propagation as it reaches the end surface. Thus at $\xi = 1.5$, the temperature response for $\eta < 0.5$ behaves as if the medium were semi-infinite, while the curves for $\eta > 0.5$ show in the influence of the surface at $\eta = 1$. It is also found that the magnitude of the temperature jump at the wave front decays with the increase of time.

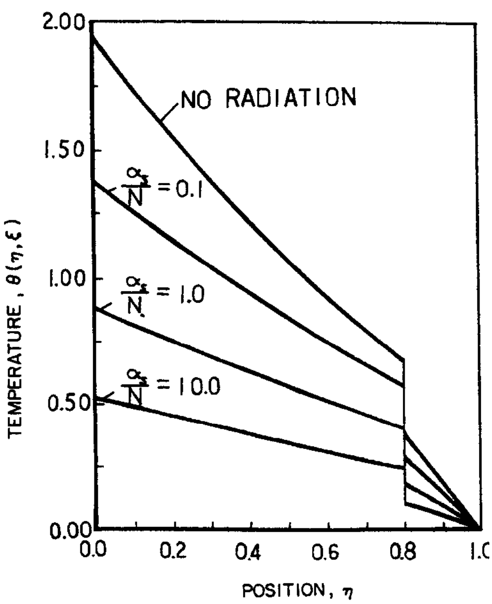

FIG. 2. Effects of surface radiation on temperature distributions at $\xi = 1.2$ in the medium with isothermal boundary at $\eta = 1$.

More numerical computations are performed in order to examine the behaviour of the wave for $\xi > 1.0$. Figures 2 and 3 show the temperature response at $\xi = 1.2$ for various values of α_s/N. As can be seen in Figs. 2 and 3, the increase of surface radiation reduces the temperature distribution in the medium and the magnitude of the temperature jump at the wave front. For the case shown in Fig. 2 the temperature at $\eta = 1$ is maintained at $\theta = 0$, so the surface at $\eta = 1$ absorbs part of the thermal energy and the temperature distributions are reduced, as shown in Fig. 2. For the case shown in Fig. 3 the surface at $\eta = 1$ is insulated so that the thermal energy propagating with the wave is reflected back to the medium. Thus the temperature distribution increases after the reflected wave front has reached the insulated surface.

In summary, the integral equation solutions show that the increase of surface radiation reduces the temperature distributions and, qualitatively, the wave nature of heat propagation, including the reflection at the end surface, is not changed by surface radiation.

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